

The collectively-tuned critical state underlies “collective minds”

Collin Feng Hu

College of Physics and Electronic Engineering, Chongqing Normal University, College Town, Shapingba District, 401331 Chongqing, China.

Abstract

Collective animal groups, the starling flocks particularly, may work at a critical state in which the efficiency of information propagation through the group is greatly increased. Here, we propose a minimum individual-based model to explore how the critical state can be reached. “Quorum response”, a type of social interaction in which an individual’s chance of making one option is very likely if the number of its local neighbors committing to this option exceeds a threshold, is introduced as the local interaction. We highlight the potential for the enhanced ability of information transfer in the group at a critical state which endows each individual the ability to access information maximally to respond to environmental perturbations. The correlation function in 3D shows a scale free correlation which is supported by the observations in field experiments.

Keywords:

collective animal behaviors, quorum response, information transfer, scale-free correlations

Email address: emergencehufeng@gmail.com (Collin Feng Hu)

1. Introduction

Collective animal groups, e.g. starling flocks, fish schools, are recently being called “collective minds”[1], because of the coherent movements and the efficient responses to environmental perturbations as if the whole group is in one mind. It is obvious that the efficiency of information transfer in the group plays a key role in enabling these abilities. Cavagana et al. observed that in the airborne motion of large starling flocks, the length of correlation between two individuals’ state scales with the size of the flock, the so called “scale-free correlation”[2]. This observation reveals that the starling flocks work at a critical state, in which one individual can effectively affect the state of any others’ no matter what the group size is, and vice versa. This property confers the group an ability to share information efficiently so that it can optimally respond to external perturbations. A pioneering study on how information transfer in a collective animal group was carried out in a fish school reacting to a risky perturbation in front[3]. It was found that the group members at the front made a quick rotation from the risk and their local neighbors behind imitated this behavior. The consecutive rotations of the group members resulted in a rapidly traveling “information waves”, which rippled from the front to the rear at a speed 10 times faster than individual fish’s speed. However, besides these experimental studies, the underlying micro-mechanism of information transfer is left largely ignored, particularly at an alarming situation in field[4]. There are some other collective systems, such as swarms of cancer cells[5], bacterial colonies[6] and even human brains[7], share many similarities to the collective animal groups, and the working efficiency of which hugely depends on the underlying mechanism of

information transfer and processing.

In this paper, we propose a minimum individual-based model to explore how the critical state of the collective animal groups, particularly the starling flocks in the airborne motion, can be reached. We apply an adapted form of “quorum response” as the local interaction rule and show that the group is poised at a critical state by tuning two parameters concerned with the distribution of individual’s vigilance. The critical state is marked by a power law distribution of the size of the “information waves” -which is measured by counting the number of the informed individuals (turned from naive ones) in one run of information propagation. This feature endows each individual the potential ability to access information maximally to respond to environmental perturbations. We also calculate the correlation function in 3D and it shows a scale-free correlation which is supported by the observations in field experiments.

2. Quorum response

Quorum response is an interaction widely found in the bee and ant colonies[8, 9], the cockroach aggregations[10], the broiler chicken crowds[11] and the fish schools[12]. The essence of quorum response is that an individual is very likely to choose one option among others if the number of its local neighbors committing to this option exceeds a threshold. Let’s consider the following simple example, suppose there are only two options, e.g. being at an alarmed state (“+” state) or a naive one (“-” state). The mathematical

description of quorum response is as follows[13]:

$$p_+^i(t+1) = \frac{\left(\frac{n_+^i(t)}{n_0 * \alpha_i}\right)^k}{1 + \left(\frac{n_+^i(t)}{n_0 * \alpha_i}\right)^k}, \quad p_-^i(t+1) = 1 - p_+^i(t+1) \quad (1)$$

where $p_+^i(t+1)$ is the probability of the individual i choosing to be at an alarmed state at time step $t+1$, n_0 is the constant total number of local neighbors to the individual i and $n_+^i(t)$ or $n_-^i(t)$ is the number of local neighbors who commits to the “+” or “-” option at time t , respectively. Obviously, the equation $n_+^i(t) + n_-^i(t) = n_0$ holds. The term $n_0 * \alpha_i$ plays the role of the threshold value for individual i . Parameters α_i , which quantifies how sensitive the individual i responds to its local neighbors’ commitments is coined as “vigilance number” below. Parameter k determines the steepness of the function. In Figure 1, we see that the probability is a monotonic increasing function. Near the threshold value, it has an inflection point with a rapid increase and the function is sigmoid. When the parameter k approaches 10, the plot is practically a step-like switch at the threshold value, jumping from zero to unity. Quorum response is essentially a distributed positive feedback process that enables information propagation. It is believed that this interaction can enhance decision speed and accuracy for a group to make a collective decision[13].

Until now, because of the complexity of the topic, how individuals in the collective animal group interact with each other in the field does not have a conclusive answer[14, 15]. For simplicity, we adapt the interaction of quorum response as a step-like form function so that the focal individual’s probability of taking an option is 0 or 1 depending on whether the number of its nearest neighbors committing to this option exceeds the threshold value of the focal

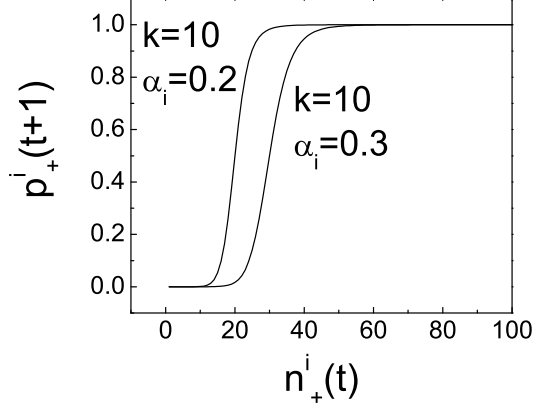


Figure 1: Function of quorum response according to equation (1). The y axis is the probability for the individual i at time step $t + 1$ to choose the option (+) and the x axis is the number of the local neighbors who commit to the option (+) at time t . The total number of local neighbors to the individual i is set to 100, $n_0 = 100$. Parameter $k = 10$ determines the steepness of the function. α_i , which quantifies the sensitivity of the individual i to the commitments of its local neighbors, is coined as “vigilance number” to the individual i . It is related to the threshold value by $n_0 * \alpha_i$.

individual.

3. A minimum model

A group is composed of N individuals, in which each one is assigned a “vigilance number” α_i . We assume that the distribution of α_i obeys a truncated Gaussian distribution in the interval $(0, 1)$, with the mean being $\bar{\alpha}$ and standard deviation being Δ . Each individual is fixed on an evenly spaced grid during the process of information propagation. The threshold

value of the individual i is defined as,

$$T_i \equiv n_0 * \alpha_i$$

where n_0 is the constant number of nearest neighbors to an individual ($n_0 = 4$ in 2D and $n_0 = 6$ in 3D in the model, with some modifications to the individuals at boundaries). Each individual can either be in an alarmed state (“+” state) $\sigma = 1$ or in a naive state (“-” state) $\sigma = -1$. The only rule of local interaction is the adapted quorum response rule:

$$\text{if } T_i < n_+^i(t) \quad \text{then } \sigma_i(t' > t) = 1$$

If the condition is satisfied, then the individual i will turn into an alarmed one $\sigma_i(t+1) = 1$ at the next time step and remains unchanged during the process of information propagation. If not satisfied, and if the individual i is at a naive state, then it will remain at a naive state $\sigma_i(t+1) = -1$ at the next step. When the individual i turned from a naive state into an alarmed one, it will continue to interact with its nearest neighbors. The local interactions may eventually cascade into “information waves” at a global level. Initially all the individuals are set to naive state (“-” state) and information is tapped into the system by randomly picking one individual at the peripheries and turning its state to the alarmed one (“+” state). A process of information transfer is considered completed when all the alarmed individuals don’t have the ability to alter the state of its nearest neighbors anymore.

We find that if the parameters of $\bar{\alpha}$ and Δ are fine tuned- $\bar{\alpha} = 0.28$, $\Delta = 0.15$ for 2D and $\bar{\alpha} = 0.29$, $\Delta = 0.35$ for 3D to be specific- a power law distribution of the size of information waves is emerged (see Figure 2). The size of the information waves is quantified by counting the number of

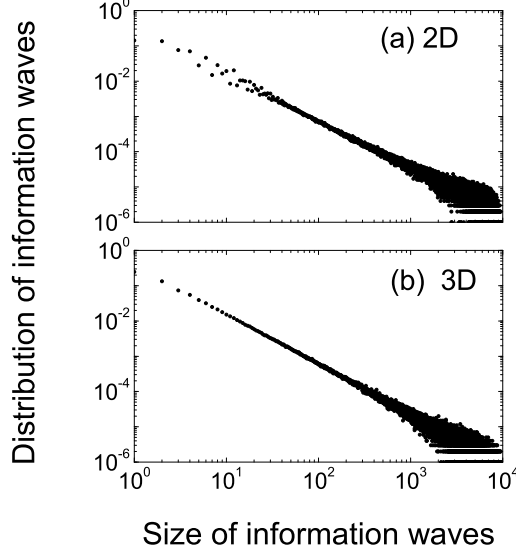


Figure 2: Distribution of the size of the information wave. (a) 2D 100×100 array. The fine tuned parameters in a Gaussian distribution of α_i are mean $\bar{\alpha} = 0.28$ and standard deviation $\Delta = 0.15$, and data is averaged over 5×10^5 simulation; (b) 3D $25 \times 25 \times 25$ array. The parameters are mean $\bar{\alpha} = 0.29$ and standard deviation $\Delta = 0.35$, and data is averaged over 5×10^5 simulation. Note the double-log of the plot.

the affected individuals who turned from naive state into alarmed one in one run of information transfer process. The curve is linear for more than three decades, with slope of the fitting line is -1.27 for 2D and -1.31 for 3D, indicates that the fine-tuned parameters have poised the system to a critical state[16]. Suppose the mean value $\bar{\alpha}$ is lower, the whole group is poised at a super-critical state, where any small perturbations will very likely to cascade into big information waves that affect a lot of individuals. It will cause a lot of energy waste. If the mean value $\bar{\alpha}$ is higher, the whole group is poised at a sub-critical state, the perturbations only affect in local areas, and

information transfer is greatly hampered. The collectively-tuned parameters of $\bar{\alpha}$ and Δ , which poise the group to a critical state, are resulted from a long-term adaptation in the risky nature[17]. In the model, for clarity, we assume the vigilance number of each individual in the group obeys a same Gaussian distribution. Yet it was observed in field that individuals at periphery are more vigilant in average than their conspecifics in center, the so called “edge effects”[18, 19]. This fact will increase the probability of the big information waves arising, which practically enhance communication between individuals[21].

We would like to make a comparison with the well-studied theory of self-organized criticality (SOC), which was proposed by Bak, Tang and Wiesenfeld in 1987[20]. It is now a commonly accepted underlying mechanism to a wide range of phenomena such as earthquakes[23], avalanches in granular materials [16]etc, and even applied to the dynamics of neural networks [25] and natural evolution[24]. It states that a complex system can organize itself to a critical state in a dynamic process without any tuning of parameters from outside. The mark of a system entering a critical state is a power law distribution of the size of avalanches- the counterpart of information wave- which is very similar to our model. However, the theory of SOC states that the self-organized critical state, which is not necessarily subject to natural selection, is a robust attractor for a complex system no matter what the initial state is. On the contrary, the critical state of our model is already collectively-tuned under the selection pressure in the risky environments.

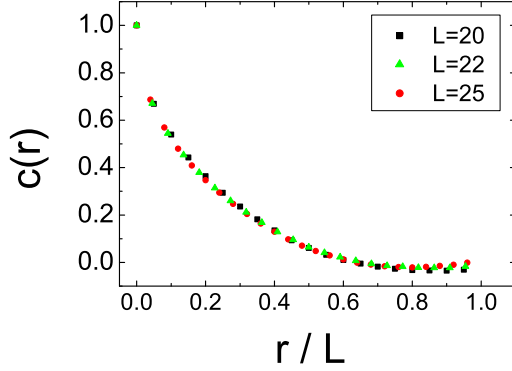


Figure 3: Correlation Versus the rescaled length. Correlation function, at a critical state, of groups with different size is plotted versus the rescaled distance r/L .

4. A scale-free correlation function

There are already observations from field experiments of the correlation function of the individuals' states in starling flocks[2]. The formula of the correlation functions is,

$$C(r) = \frac{1}{c_0} \frac{\sum_{ij} \sigma_i \sigma_j \delta(r - r_{ij})}{\sum_{ij} \delta(r - r_{ij})}$$

which states how strength of the influence from one bird to any other one varies with the distance r . Delta function $\delta(r - r_{ij})$ picks a pair of individual i and j at a distance of r . c_0 is a normalization factor such that $C(r = 0) = 1$. In the field experiments of starling flocks, it was found there are two large clusters of strongly correlated birds in the flock, where individuals in each cluster have the same velocity state. In order to mimic this condition, we only calculate the correlation function when a sufficient large information wave ($> 0.3 \cdot N$) is propagated so that the group is divided into two clusters

of alarmed individuals and naive ones at a global level. In each simulation 50 individuals are randomly sampled and the correlation function is averaged among the sample. The distance was calculated in discrete integers, $r_{ij} = \lfloor \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \rfloor$, where $\lfloor \cdot \rfloor$ is the floor function. We find that when the system is tuned into a critical state the correlation function of the group at different size collapse together (see Figure 3), which shows that two individuals are correlated in a way that the correlation strength is not relevant to the group size, a scale-free correlation.

Recently, many studies focus on how the ordered state is resulted from local interactions and how the directional information is propagated through the group. Theoretical [26, 4] and experimental [27] studies have revealed that it is a universal property of a collective animal group to evolve into an ordered state by applying “averaging the velocity among its local neighbors” as the interaction rule. By this way the informed individual(s) has to interact with the naive one(s) repeatedly to spread the directional information[4]. It is a gradual process, which is very unlikely that the alarming information is propagated this way. On the contrary, applying the interaction rule of quorum response and positioning the system at a critical state, individuals can respond to the risky environments in a drastic way by cascading a local perturbation to a big information wave. Thus information is propagated more efficiently.

5. Conclusions

We proposed a minimum model, with introduction of an adapted form of quorum response as the local interaction rule among individuals, to study

how information is propagated in the collective animal group. By tuning the parameters of the mean and the standard deviation of a Gaussian distribution of individual's "vigilance number", we found that the group is poised to a critical state. It is reflected by a power law distribution of the size of the information waves. Being poised at a critical state, each individual can access information maximally to respond to environmental perturbations. We also calculated the correlation function of different sized groups in 3D and found that data collapsed on one curve. This demonstrates that the correlation is scale free, which is supported by the observations in field experiments.

We hope this minimum model captures the essences of information transfer in the collective animal group and highlights the advantages of critical state underlying information transfer. Recently, it was found that the cortical neuron networks work at a critical state, on which the information transmission, adaptability and computational power would be optimized[21, 25]. It seems the "collective minds" shares deep similarities to "minds" indeed.

References

- [1] I.D. Couzin, Collective minds, *Nature*, **445**, 715 (2007).
- [2] A. Cavagna, A. Cimorelli, I. Giardina, G. Parisi, R. Santagati, F. Stefanini, and M. Viale, Scale-free correlations in starling flocks, *PNAS* **107**, 11865 (2010).
- [3] D.V. Radakov, Schooling in the ecology of fish, (John Wiley and Sons, New York, 1973).
- [4] I.D. Couzin, J. Krause, N.R. Franks, and S.A. Levin, Effective leadership

- and decision-making in animal groups on the move, *Nature* **433**, 513 (2005).
- [5] T.S. Deisboeck, and I.D. Couzin, Collective behavior in cancer cell populations, *BioEssays*, **31**, 190 (2009).
 - [6] H.P. Zhang, A. Beer, E.-L. Florin, and H.L. Swinney, Collective motion and density fluctuations in bacterial colonies, *PNAS*, **107**, 13626 (2010).
 - [7] J.J. Hopfield, Neurons, Dynamics and Computation, *Physics Today* **47**, 40 (1994).
 - [8] T.D. Seeley, and P.K. Visscher, Quorum sensing during nest-site selection by honeybee swarms, *Behav. Ecol. Sociobiol.* **56**, 594 (2004).
 - [9] S.C. Pratt, E.B. Mallon, D.J.T. Sumpter, and N.R. Franks, Quorum sensing, recruitment, and collective decision-making during colony emigration by the ant *Leptothorax albipennis*, *Behav. Ecol. Sociobiol.* **52**, 117 (2002).
 - [10] Amé, J.M., Halloy, J., Rivault, C., Detrain, C., Deneubourg, J.L., Collegial Decision Making Based on Social Amplification Leads to Optimal Group Formation. *PNAS* **103**, 5835 (2010).
 - [11] L.M. Collins, and D.J.T. Sumpter, The feeding dynamics of broiler chickens, *J. R. Soc. Interface* **4**, 65 (2007).
 - [12] A.J.W. Ward, D.J.T. Sumpter, I.D. Couzin, P.J.B. Hart, and J. Krause, Quorum decision-making facilitates information transfer in fish shoals, *PNAS* **105**, 6948 (2008).

- [13] D.J.T. Sumpter, and S.C. Pratt, Quorum response and consensus decision making, *Phil. Trans. R. Soc. B.* **364**, 743 (2009).
- [14] J.E. Herbert-Read, A. Perna, R.P. Mann, T.M. Schaerf, D.J.T. Sumpter and A.J.W. Ward, Inferring the rules of interaction of shoaling fish, *PNAS* **108** 18726 (2011).
- [15] Y. Katz, K. Tunstrøm, C.C. Ioannou, C. Huepe, and I.D. Couzin, Inferring the structure and dynamics of interactions in schooling fish, *PNAS* **108** 18720 (2011).
- [16] P. Bak, *how nature works: the science of self-organized criticality* (Springer-Verlag Inc. , New York 1996). The paradigm of the SOC theory is the sandpile model. Imagine a table where sand is slowly dropped is represented by a two dimensional lattice, and each grid can hold up to three grains of sand. When the fourth grain is added, the site is driven over threshold and topples, shifting grains to each of its four nearest neighbors. If any of these neighboring grids is over threshold, it too will re-distribute sand to its neighbors. A process initiated from a local perturbation may repeat this way to cascade to affect many grids, which is called an avalanche. The “finger print” when the system enters into a critical state is a power law distribution of the size of the avalanches.
- [17] T. Mora and W. Bialek, Are Biological Systems Poised at Criticality?, *J. Stat. Phys.* **144** 268 (2011).
- [18] G.C. Keys, and L.A. Dugatkin, Flock size and position effects on vig-

- ilance, aggression, and prey capture in the European starling, *The Condor* **92**, 151 (1990).
- [19] P. Blanchard, R. Sabatier, and H. Fritz, Within-group spatial position and vigilance: a role also for competition? The case of impalas (*Aepyceros melampus*) with a controlled food supply, *Behav. Ecol. Sociobiol.* **62**, 1863 (2008).
 - [20] P. Bak, C. Tang, and K. Wiesenfeld, Self-Organized Criticality: An explanation of $1/f$ noise, *Phys. Rev. Lett.* **59**, 381 (1987).
 - [21] J.M. Begs, The criticality hypothesis: how local cortical networks might optimize information processing, *Phil. Trans. R. Soc. A* **366**, 329-343 (2008).
 - [22] X.W. Zhao, and T.L. Chen, Type of self-organized criticality model based on neural network, *Phys. Rev. E* **65**, 026114-1-024116-6 (2002).
 - [23] P. Bak, K. Christensen, L. Danon and T. Scanlon, Unified Scaling Law for Earthquakes, *Phys. Rev. Lett* **88**, 178501-1-17805-4 (2002).
 - [24] P. Bak and K. Sneppen, Punctuated equilibrium and criticality in a simple model of evolution, *Phys. Rev. Lett* **71**, 40803-40806 (1993).
 - [25] X.W. Zhao, and T.L. Chen, Type of self-organized criticality model based on neural network, *Phys. Rev. E* **65**, 026114-1-024116-6 (2002).
 - [26] T. Vicsek, A. Czirok, E.B. Jacob, I. Cohen, and O. Shochet, Novel Type of Phase Transition in a System of Self-Driven Particles, *Phys. Rev. Lett.* **75**, 1226 (1995).

- [27] J. Buhl, D.J.T. Sumpter, I.D. Couzin, J.J. Hale, E. Despland, E.R. Miller, and S.J. Simpson, From disorder to order in marching locust, Science **312**, 1402 (2006).